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A General Model for the Calculation of Daylighting in Interior Spaces

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An analytical model and a computer code have been developed which calculate the amount of daylight illumination on a working surface inside an arbitrary room, for overcast as well as for clear sky conditions. The room may have windows as well as skylights, with clear glass, diffusing glass, or glass fitted with thin shading devices (such as sheer curtains or shades), as well as overhangs. The shape of the room is not limited to simple, rectangular enclosures, allowing the treatment of L-shaped rooms, A-frame buildings, etc.

The illumination generally consists of three parts: direct sky illumination, illumination from external reflectors, and illumination from internal reflectors. First, the luminances emanating from surrounding obstructions are determined. Next, illumination traveling through the windows directly to inside walls and working surface is calculated. Finally, interreflection inside the room is taken into account to establish the luminance distributions of inside walls. After determination of all inside and outside luminances, it is a simple matter to calculate illumination and daylight factor for the working surface.

INTRODUCTION

When designing a room one of the considerations taken into account is the amount of daylight illumination that enters the room. To maximize human efficiency and comfort it is often desirable to optimize daylight distribution within the room on a specified working surface. Simultaneously, it is desirable to optimize window surface areas and orientation to minimize heating and cooling loads in the light of energy conservation.

Daylight illumination falling onto a working surface consists of three components: (i) light traveling directly from the sky through windows to the working surface, (ii) light traveling directly from outside reflectors (such as opposing buildings, ground, etc.) through the windows to the working surface, and (iii) light traveling to the working surface after one or more reflections from inside surfaces (walls, ceiling, etc.).

Predictions of daylight illumination levels in a room are, by necessity, subject to a compromise between accuracy and numerical complexity. If only crude knowledge of general illumination levels is needed, simple models, such as the one by Bryan [1], which do not require use of a digital computer, may be sufficient. For more accurate evaluations the code developed by DiLaura *et al.* [2 - 5] represents the state of the art, at the expense of substantial computer time requirements. Nevertheless, even the sophisticated model by DiLaura *et al.* is subject to a number of confining restrictions: (i) only rectangular rooms with horizontal and vertical rectangular surfaces can be modeled; (ii) the room may not have any internal obstructions; (iii) internal reflections as well as window overhangs are modeled in a very approximate fashion. The above shortcomings are dictated by the need to keep computational time within reasonable bounds. It should be kept in mind here that DiLaura's is a general lighting code, of which daylighting is only one element.

It is the purpose of the present paper to develop a general and relatively simple yet accurate and efficient model specifically for the prediction of daylight illumination. In particular, the restrictions mentioned above will be eliminated from the present analysis.

SKY LUMINANCE VARIATION

To determine the contributions of direct sky illumination, illumination from external reflections, and from internal reflections, the luminance distribution over the sky must be

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known for all directions. The luminance, L, is defined as the light flux traveling into a given direction per unit area normal to the rays and per unit solid angle. The sky luminance distribution is dependent upon weather conditions, geographical location, the time of day, and the time of year, and is presumed to be known. The distribution functions used for the sample calculations in the present paper are the ones proposed by CIE (Commission Internationale de l'Eclairage) for overcast and clear days [6]. However, the model presented here may be used with any given sky luminance distribution.

GEOMETRIC MODELLING OF INSIDE AND OUTSIDE SURFACES

The inside of the room is assumed to consist of N plane surfaces of trapezoidal shape (see Fig. 1). This is considered to be

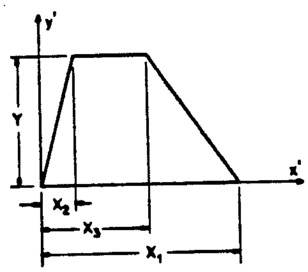

Fig. 1. Geometry of allowable surfaces.

adequately general to model any present room design of practical importance. These N surfaces comprise N_c clear windows, N_{sc} clear windows with sheer curtains, N_d diffusing windows, and N_w opaque walls, which reflect light diffusely. Clear windows are understood to be surfaces that partially transmit light without directional scattering. Diffuse windows, on the other hand, are assumed to scatter transmitted light equally into all directions (milky-texture glass, windows with shades, etc.). Sheer-curtain windows are assumed to partially transmit light directly, and to partially diffuse the light (dirty windows, fly screens, sheer-curtained windows, etc.). Each of the opaque surfaces may have other surfaces as cut-outs, *e.g.*, windows, large dark pictures, etc.

Location and dimensions of each surface are described by a local coordinate system which is then related to an overall stationary coordinate system. The local coordinate system (see Fig. 1) has its origin located so that the x'-axis runs along one of the two parallel sides of the trapezoid. The z'-axis is chosen so that it points perpendicularly into the room.

The overall stationary coordinate system is chosen in the following manner:

(i) arbitrary fixed origin,

(ii) x-axis pointing from origin towards south,

(iii) y-axis pointing from origin towards east,

(iv) z-axis pointing from origin vertically into the sky (zenith).

To totally describe a surface "i" the local coordinate system must be related to the overall coordinate system. To accomplish this the following data are required:

(i) location (X_{oi}, Y_{oi}, Z_{oi}) of the local coordinate system's origin with respect to the overall origin;

(ii) β_{xi} and β_{yi}, that is, the polar angles formed by the x'- and y'-axes with respect to the absolute z-axis;

(iii) ψ_{xi} and ψ_{yi}, that is, the azimuth angles of the x'- and y'-axes in the x–y plane (the angles between the x-axis and the projection of the x'- or y'-axis);

(iv) X_{1i}, X_{2i}, X_{3i} and Y_i, that is, characteristic dimensions of surface "i" as depicted in Fig. 1;

(v) if the surface is a window, its thickness, that is, the width of the hole through which light can penetrate (d_i).

The overall coordinates of any point (x', y') on a surface A_i can then be described by the equations

$$
\begin{aligned}
x_i &= X_{oi} + l^i_{11}x' + l^i_{12}y', \\
y_i &= Y_{oi} + l^i_{21}x' + l^i_{22}y', \qquad (1)\\
z_i &= Z_{oi} + l^i_{31}x' + l^i_{32}y',
\end{aligned}
$$

which are subject to the restrictions,

$$
\frac{x_{2i}}{Y_I}y' < x' < X_{1i} - \frac{X_{1i} - X_{3i}}{Y_i}y', \qquad (2)
$$

$$
0 < y' < Y_i.
$$

The values l^i_{mn} in eqn. (1) are the directior cosines between the m-axis of the overal system and the n-axis of the local system

They are computed from the following equations:

$$
\begin{aligned}
l_{11}^i &= \hat{\imath} \cdot \hat{\imath}_i = \sin\beta_{xi}\cos\psi_{xi},\\
l_{21}^i &= \hat{\jmath} \cdot \hat{\imath}_i = \sin\beta_{xi}\sin\psi_{xi},\\
l_{31}^i &= \hat{k} \cdot \hat{\imath}_i = \cos\beta_{xi},\\
l_{12}^i &= \hat{\imath} \cdot \hat{\jmath}_i = \sin\beta_{yi}\cos\psi_{yi}.\\
l_{22}^i &= \hat{\jmath} \cdot \hat{\jmath}_i = \sin\beta_{yi}\sin\psi_{yi},\\
l_{32}^i &= \hat{k} \cdot \hat{\jmath}_i = \cos\beta_{yi},\\
l_{13}^i &= \hat{\imath} \cdot \hat{k}_i = \sin\beta_{xi}\sin\psi_{xi}\cos\beta_{yi} - \\
&\quad - \cos\beta_{xi}\sin\beta_{yi}\sin\psi_{yi},\\
l_{23}^i &= \hat{\jmath} \cdot \hat{k}_i = \cos\beta_{xi}\sin\beta_{yi}\cos\psi_{yi} - \\
&\quad - \sin\beta_{xi}\cos\psi_{xi}\cos\beta_{yi},\\
l_{33}^i &= \hat{k} \cdot \hat{k}_i = \sin\beta_{xi}\sin\beta_{yi}\sin(\psi_{yi} - \psi_{xi}).
\end{aligned} \tag{3}
$$

Equations (1) and (2) may be rewritten in nondimensional coordinates

$$\eta = \frac{y'}{Y_i}; \ \xi = \frac{x' - X_{2i}\eta}{X_{1i} - (X_{1i} - X_{3i} + X_{2i})\eta} \tag{4}$$

such that

$$
\begin{aligned}
x_i &= X_{oi} + l_{11}^i[X_{1i} - (X_{1i} - X_{3i} + X_{2i})\eta]\xi + \\
&\quad + (l_{11}^i X_{2i} + l_{12}^i Y_i)\eta,\\
y_i &= Y_{oi} + l_{21}^i[X_{1i} - (X_{1i} - X_{3i} + X_{2i})\eta]\xi + \\
&\quad + (l_{21}^i X_{2i} + l_{22}^i Y_i)\eta,\\
z_i &= Z_{oi} + l_{31}^i[X_{1i} - (X_{1i} - X_{3i} + X_{2i})\eta]\xi + \\
&\quad + (l_{31}^i X_{2i} + l_{32}^i Y_i)\eta,
\end{aligned} \tag{5}
$$

restricted by

$$0 \leqslant \eta \leqslant 1, \qquad 0 \leqslant \xi \leqslant 1. \tag{6}$$

An "enclosure" is assigned to each window in the room (unless the window is a skylight which sees only the sky). Windows that are cutouts located in the same wall may view the same enclosure. All surfaces in the outside enclosure are assumed to be plane and of trapezoidal shape as is the case for inside surfaces. For outside surfaces, no cutouts are allowed (for example, a building facade with windows is assigned an overall reflectivity), and all surfaces are either diffuse and opaque or part of the sky. The surfaces in the enclosure are described in the same manner as those inside the room, that is, each surface is assigned a local coordinate system and that system is then related to the overall coordinate system. Thus, Fig. 1 and eqns. (1) - (6) hold for both inside and outside surfaces.

DETERMINATION OF THE LUMINANCE DISTRIBUTION ON EXTERNAL SURFACES

The luminance of all surfaces in the outside enclosures must be determined in order to calculate the amount of illumination reflected into the room from the surroundings. For simplicity it is assumed that each outside surface has a constant (average) luminance over its entire surface area. A light flux balance on each opaque surface yields (following thermal radiation procedures, *e.g.*, ref. 7)

$$\sum_{j=1}^{N_{\mathrm{wp}}} \left[\frac{\delta_{ij}}{\rho_{j\mathrm{p}}} - F_{i-j} \right] L_{j\mathrm{p}} = L_z \mathcal{F}_{i-\mathrm{sky}} + \frac{f_{i\mathrm{p}}}{\pi} E_s \cos\beta_{si}, \quad i = 1, N_{\mathrm{wp}} \tag{7}$$

where, N_{wp} = number of opaque walls in enclosure p, $L_{j\mathrm{p}}$ = unknown luminance of surface j, $\rho_{j\mathrm{p}}$ = reflectivity of surface j, F_{i-j} = light exchange factor from surface "i" to "j", $\mathcal{F}_{i-\mathrm{sky}}$ = light exchange factor from surface "i" to sky, L_z = sky luminance at zenith, E_s = direct solar illumination perpendicular to the rays, $f_{i\mathrm{p}}$ = fraction of surface $A_{i\mathrm{p}}$ receiving direct sunshine, β_{si} = angle between surface normal $\hat{k}_{i\mathrm{p}}$ and unit vector $\hat{r}_s$ pointing toward the sun ($\cos\beta_{si} = \hat{k}_{i\mathrm{p}} \cdot \hat{r}_s$), δ_{ij} = Kronecker delta.

Equations (7) form a system of N_{wp} equations in N_{wp} unknown luminances, $L_{j\mathrm{p}}$. This set of simultaneous equations may be solved for each outside enclosure by matrix inversion. Once the luminances, $L_{j\mathrm{p}}$, are known, their contribution to illumination in the room, either by direct travel or after internal reflections, may be evaluated.

The necessary light exchange factors, F_{i-j} and $\mathcal{F}_{i-\mathrm{sky}}$, may be evaluated by applying radiative transfer theory. The amount of light flux from surface "j" onto surface "i" is [7]:

$$E_{ij} = \int_{A_i} \int_{\Omega_j} L_j(\omega) \cos\beta_{ij} \mathrm{d}\omega \, \mathrm{d}A_i, \tag{8}$$

where L_j is the luminance emanating from surface "j" and β_{ij} is the angle between the surface normal to A_i and the direction under consideration, while Ω_j is the solid angle with which surface A_j is seen from a point on A_i. If L_j = const, *i.e.*, the luminance does not vary over surface A_j, eqn. (8) reduces to [7]

$$E_{ij} = L_j F_{i\to j} A_i = A_i L_j \frac{1}{A_i} \int_{A_i} \int_{A_j} \delta(\mathrm{d}A_i \to \mathrm{d}A_j) \times \\ \times \frac{\cos\beta_{ij}\cos\beta_{ji}}{\pi S_{ij}^2}\, \mathrm{d}A_j\, \mathrm{d}A_i, \tag{9}$$

where $F_{i\to j}$ is the standard configuration factor between diffuse surfaces if there are no visual obstructions. In eqn. (9) δ is an on–off function, *i.e.*, $\delta = 1$ for pairs of points on A_i and A_j that see each other, and $\delta = 0$ for pairs that do not (due to visual obstructions), while S_{ij} is the distance between the pairs.

If $L_j \neq$ const, *e.g.*, variable luminance across the sky (A_j is the sky), the $L_{\rm sky}$ cannot be separated from the integral, but one may write

$$E_{i\,\rm sky} = L_z \mathscr{F}_{i\,\rm sky} A_i \\ = A_i L_z \frac{1}{A_i} \int_{A_i} \int_{\Omega_{\rm sky}} \left(\frac{L_{\rm sky}}{L_z}\right) \cos\beta_{i\,\rm sky}\, \mathrm{d}\omega\, \mathrm{d}A_i, \tag{10}$$

where L_z is the zenith luminance.

The exchange factors can be evaluated in a number of ways. If the surface luminance is constant and if there are no visual obstructions, eqn. (9) is readily evaluated by double area integration or, after transformation, by contour integration. This method, however, becomes impractical for the more general cases.

For the relatively few and large outside surfaces the Monte Carlo method [7] is used to determine the $F_{i\to j}$ and $\mathscr{F}_{i\to \rm sky}$, which is considered most efficient for these cases. In this statistical numerical method a large number of light bundles is traced and their behavior averaged. Note in this context that eqn. (10) is also an expression for the light flux traveling from surface "i" into the sky, if A_i had a directional distribution of luminance identical to the sky's. Thus, for the evaluation of all exchange factors, locations and directions of light emission on A_i are chosen by picking four random numbers R_1 to R_4 ($0 \leqslant R_i \leqslant 1$), resulting in [7]

$$\frac{y_i'}{Y_i} = \frac{X_{1i} - \sqrt{X_{1i}^2 - [X_{1i}^2 - (X_{3i} - X_{2i})^2] R_1}}{X_{1i} - (X_{3i} - X_{2i})}; \quad X_{1i} \neq X_{3i} - X_{2i}$$
$$= R_1; \quad X_{1i} = X_{3i} - X_{2i}, \tag{11}$$

$$x_i' = X_{2i}\frac{y_i'}{Y_i} + \left[X_{1i} - (X_{1i} - X_{3i} + X_{2i})\frac{y_i'}{Y_i}\right] R_2, \tag{12}$$

$$\beta = \sin^{-1}\sqrt{R_3}, \tag{13}$$

$$\theta = 2\pi R_4. \tag{14}$$

The above information is used to form a unit vector, $\hat{r}$, for the direction of emission, whose components in the overall coordinate system are

$$\hat{r}\cdot\hat{i} = n_1 = l_{11}^i \sin\beta\cos\theta + l_{21}^i \sin\beta\sin\theta + \\ + l_{31}^i \cos\beta$$
$$\hat{r}\cdot\hat{j} = n_2 = l_{12}^i \sin\beta\cos\theta + l_{22}^i \sin\beta\sin\theta + \\ + l_{32}^i \cos\beta$$
$$\hat{r}\cdot\hat{k} = n_3 = l_{13}^i \sin\beta\cos\theta + l_{23}^i \sin\beta\sin\theta + \\ + l_{33}^i \cos\beta. \tag{15}$$

Once the point of emission and the direction vector have been determined, the surface upon which the light bundle impinges must be found. If $\vec{r}_i$ represents the vector to the point of emission on surface "i", and $\vec{r}_j$ represents the vector to a possible point of intersection on surface "j", the local coordinates of this intersection point can be computed from:

$$\frac{(\vec{r}_j - \vec{r}_i)\cdot\hat{i}_j}{\hat{r}\cdot\hat{i}_j} = \frac{(\vec{r}_j - \vec{r}_i)\cdot\hat{j}_j}{\hat{r}\cdot\hat{j}_j} = \frac{(\vec{r}_j - \vec{r}_i)\cdot\hat{k}_j}{\hat{r}\cdot\hat{k}_j} = d, \tag{16}$$

where d is the distance traveled.

Equation (16) yields

$$x_j' = (x_i - X_{oj} + an_1) l_{11}^j + (y_i - Y_{oj} + an_2) l_{21}^j + \\ + (z_i - Z_{oj} + an_3) l_{31}^j,$$
$$y_j' = (x_i - X_{oj} + an_1) l_{12}^j + (y_i - Y_{oj} + an_2) l_{22}^j + \\ + (z_i - Z_{oj} + an_3) l_{32}^j, \tag{17}$$

where

$$a = \frac{(x_i - X_{oj}) l_{13}^j + (y_i - Y_{oj}) l_{23}^j + (z_i - Z_{oj}) l_{33}^j}{n_1 l_{13}^j + n_2 l_{23}^j + n_3 l_{33}^j}. \tag{18}$$

The restriction by eqn. (2) is now applied to eqns. (17) to see if (x_j', y_j') is actually on surface "j". All surfaces are checked in this manner until the correct intersection (with minimum d) is found. More than one intersection may be possible as some surfaces may be partially obstructed by other surfaces (overhangs, balconies, etc.).

A tally is kept of the percentage of light bundles that hits each surface, which is the exchange factor. If the intersecting surface is the sky, it can be shown that the exchange factor defined in eqn. (18) is determined by multiplying each bundle by the weight-factor L_{sky}/L_z.

DETERMINATION OF THE LUMINANCE DISTRIBUTION ON INTERNAL SURFACES

Inside the room luminances may not only vary significantly across the surfaces, but this variation may also profoundly affect the illumination on the working surface. It is, therefore, necessary to break up inside surfaces into a number of subsurfaces or nodes. A light balance performed on subsurface "k" on surface "i" yields (again using radiative transfer analogy):

$$L_{ik} = \rho_i \sum_{j=1}^{N} \sum_{l=1}^{N_j} F_{ik\to jl} L_{jl} + \rho_i L_{cik} + L_{di}, \quad \left\{ \begin{array}{l} i = 1, N \\ k = 1, N_i \end{array} \right\}, \tag{19}$$

where L_{cik} is the directly transmitted luminance penetrating through a clear or sheer-curtained window (as seen by node "k"), and L_{di} is the contribution of diffusing or sheer-curtained windows to their own luminance. Thus incoming light is "assigned" to the surface where the first diffuse reflection or diffuse re-orientation after transmission occurs, *i.e.*, some inside surface for light coming through clear windows, and the window itself if it is diffusing. Windows with simple shading devices, such as sheer curtains, are assumed to transmit a fraction, α, like a clear window, while the rest, $1 - \alpha$, is diffused. This modelling allows the usage of standard radiation configuration factors, $F_{ik\to jl}$ [7].

It follows that

$$L_{cik} = \sum_{j=1}^{N_c+N_{sw}} \alpha_j \sum_{l=1}^{N_j} F_{ik\to jl} \tau_{ik-jl} L_p + \frac{\delta_c}{\pi} \tau_{ik-s} E_s \cos\beta_{sik}. \tag{20}$$

In this equation L_p is the luminance of that outside surface (or the sky), from which a light beam can travel along a straight line through node "jl" to the node under consideration, "ik". The transmissivity of the window in this direction is denoted by τ_{ik-jl}. The last term specifies whether direct sunshine falls on the inside node ($\delta_c = 1$) or not ($\delta_c = 0$).

Also

$$L_{di} = (1-\alpha_i)\left[\int_{2\pi} L_p(\omega)\tau_i(\omega)\cos\beta\, d\omega + \frac{\delta_c}{\pi}\tau_{i-s}E_s\cos\beta_{si}\right], \quad N_c < i \leqslant N_c + N_{sw} + N_d \tag{21}$$

where the integration is over the entire outside hemisphere from which light can impinge on the diffusing window.

For the numerous small subsurfaces inside the room the exchange factors are readily determined from eqn. (9) as

$$F_{ik\to jl} = \frac{1}{A_{ik}} \int_{A_{ik}} \int_{A_{jl}} \delta(dA_{ik} \to dA_{jl}) \times \frac{\cos\beta_{ij}\cos\beta_{ji}}{\pi S_{ij}^2} dA_{jl}\, dA_{ik}. \tag{22}$$

If the nodes are sufficiently small and sufficiently far apart, say $A_{ik}A_{ji}/S_{ij}^4 < 1$, the integrand in eqn. (22) may be replaced by a constant average value evaluated between node centers, resulting in

$$F_{ik\to jl} \cong \left[\delta(A_{ik} - A_{jl}) \frac{\cos\beta_{ij}\cos\beta_{ji}}{\pi S_{ij}^2}\right]_{avg.} A_{jl}; \quad \frac{A_{ik}A_{jl}}{S_{ij}^4} < 1. \tag{23}$$

For the few nodes close to each other (touching corners) eqn. (22) is readily evaluated by numerical quadrature.

Equation (19) again forms a system of simultaneous equations. However, depending on the number of nodes, this sytem may have many hundreds of equations, making matrix inversion impractical. As the major contribution to L_{ik} is generally due to direct illumination (L_{cik} and L_{di}), eqn. (19) is readily solved by successive approximations (one or two iterations usually suffice).

DIRECT SUNSHINE DURING CLEAR SKY CONDITIONS

When the sun is not occluded by clouds, direct sunshine falls on some of the opaque surfaces of the outer enclosures and, possibly, directly into the room through windows. For each outside surface the fraction of it, f_i, that receives direct sunlight must be calculated. This value is then used in eqn. (7). Similarly, for each inside node it must be determined whether it receives sunshine ($\delta_c = 1$) or not ($\delta_c = 0$), to be used in eqns. (20) and (21). Finally, if an outside surface, A_i, can be seen from the working surface inside the room so that direct reflection from A_i is possible, then unshaded areas upon A_i must be identified so that their contribution to the working surface illumination can be determined.

If unobstructed sunlight shines on the surface A_i, the unshaded fraction of A_i can be evaluated from:

$$f_i = \frac{1}{A_i} \int_{A_i} \delta_i \, dA_i \tag{24}$$

where

$$\delta_i = 0 \text{ for shaded regions and,} \qquad \delta_i = 1 \text{ for unshaded regions.} \tag{25}$$

Equation (24) may be written as

$$f_i = \frac{1}{A_i} \int_0^Y \int_{X_l(y_i)}^{X_r(y_i)} \delta_i(x_i, y_i) \, dx_i \, dy_i \tag{26}$$

where, referring to Fig. 1,

$$X_l(y_i) = X_{2i}(y_i/Y_i), \qquad X_r(y_i) = X_{1i} - (X_{1i} - X_{3i})y_i/Y_i. \tag{27}$$

Equation (26) may be written in terms of the nondimensional parameters η and ξ, defined by eqns. (5), as

$$f_i = \frac{2}{(X_{1i} + X_{3i} - X_{2i})} \int_0^1 \int_0^1 \delta(\eta, \xi)[X_{1i} - (X_{1i} - X_{3i} + X_{2i})\eta] \, d\eta \, d\xi. \tag{28}$$

The evaluation on this integral may be achieved by numerical quadrature.

To determine whether a point (η, ξ) is shaded or unshaded, a light beam is traced from that point into the direction of the sun, $\hat{r}_s$, by applying eqns. (15) - (18).

The determination of the δ function for inside nodes is similar: a beam from that node traveling towards the sun is checked whether it passes through a window without hitting any inside or outside obstructions.

WORKING SURFACE ILLUMINATION AND DAYLIGHT FACTOR

By placing one or more "imaginary working surfaces" into the room, illumination at points inside the room where no surface is located can be calculated.

Each working surface is divided into a grid of nodal points. The illumination at any working surface node "ik", in the room can be computed similar to eqn. (19) from

$$E_{ik} = \pi \sum_{j=1}^{N} \sum_{l=1}^{N_j} L_{jl} F_{ik \to jl} + \pi L_{cik}. \tag{29}$$

where L_{cik} is again calculated from eqn. (20).

The daylight factor is calculated from

$$\mathrm{DF} = E_{ik}/E_{\mathrm{H}}, \tag{30}$$

where E_{H} is the illumination onto an unobstructed (outside) horizontal surface. In the generally accepted definition of the daylight factor, direct sunshine is specifically excluded from E_{ik} as well as E_{H}. Note that this definition may result in DF > 1 (due to reflection from surfaces receiving direct sunshine).

Recall that for all outside surfaces the contribution of direct sunshine is averaged over the whole surface when computing the surfaces' overall luminance. However, to accurately compute the contribution of outside opaque surfaces to the illumination of the working surface nodal points, unshaded areas of outside surfaces must be differentiated from shaded areas. This is accomplished in the following manner: when the fraction of direct sunshine on an outside surface is computed by numerical integration, the nodal points on the surface are checked to see if they receive direct sunshine or not, and this fact is then stored. Then, when computing the outside luminance, L_p, in eqn. (20), the location intersected on the outside wall by the light beam is checked to see if it is in an unshaded region or not. For an intersection in an unshaded region, the luminance of outside surface "i" is evaluated as

$$L_P = L_i + (1 - f_i) \frac{\rho_i}{\pi} E_s \cos \beta_{si}, \tag{31a}$$

and for a shaded region as

$$L_P = L_i - f_i \frac{\rho_i}{\pi} E_s \cos \beta_{si}, \tag{31b}$$

where f_i is the fraction of the outside surface that sees direct sunshine, ρ_i is the reflectivity of the outside surface, L_i is the average luminance on the outside surface and E_s is the direct sky illumination.

ILLUSTRATIVE EXAMPLES

To demonstrate the power of the present model, two different room designs will be considered. In the first example daylighting in an L-shaped room is considered, *i.e.*, a room where some walls are partially obstructed

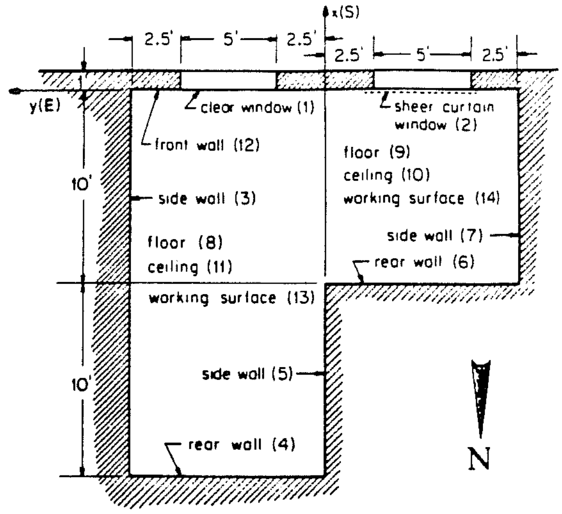

Fig. 2. Floor plan of L-shaped room.

from one another (see Fig. 2). In the second example a one-room A-frame space with window overhangs is treated, *i.e.*, a room with non-rectangular surfaces. Design data for the two rooms and their surroundings are summarized in Table 1.

The results for the daylight illumination upon a 2.5 ft. (0.76 m) high working surface in the L-shaped room are summarized in Table 2 for an overcast day, and in Table 3 for a clear day. As the sky luminance for a CIE overcast sky is symmetric, illumination inside the room would be symmetric as well, if there were a thin, opaque separation wall in

TABLE 1

Design data for sample rooms

	L-shaped room	A-frame room
Window number and size	Two 5 ft × 5 ft (1.5 m × 1.5 m) windows in south wall, centers 5 ft (1.5 m) above floor, 1 window with sheer curtain (α = 0.8)	2 triangular windows 15 ft (4.57 m) (base) × 15 ft (1.57 m) (height) filling entire east and west walls
Floor height above ground	30 ft (9.15 m)	0 ft (0 m)
Window transmissivities (perpendicular to windows)	85%	85%
Reflectivities: floor	20%	20%
ceiling	70%	—
sidewalls	60%	30%
Room height	10 ft (3.05 m)	15 ft (4.57 m)
Window wall/overhang width	1 ft (0.30 m)	0, 1 or 3 ft (0, 0.30 or 0.91 m)
Object building and surroundings	Room in center of 60 ft × 30 ft × 50 ft (18.3 m × 9.2 m × 15.2 m) building. Windows in 60 ft (18.3 m) E–W wall exposed to the South; identical opposing building to the North, with 50 ft (15.2 m) between buildings	No outside obstructions

TABLE 2

Working surface illumination in L-shaped room on an overcast day (in foot-candles = 10.76 lux) Illumination on outside horizontal surface: 1400 fc (15064 lux).

x	y: 8.75	6.25	3.75	1.25	−1.25	−3.75	−6.25	−8.75
−1.25	51.3	205.6	205.8	54.1	54.0	200.1	200.9	52.1
−3.75	61.9	98.7	101.2	70.3	69.3	98.9	97.5	63.4
−6.25	36.4	43.5	45.7	45.9	48.3	50.7	49.4	41.8
−8.75	23.1	26.3	28.1	30.0	36.5	38.1	36.3	31.6
−11.25	19.3	21.9	23.1	21.6				
−13.75	14.5	15.7	15.7	13.0				
−16.25	12.2	12.9	12.2	10.3				
−18.75	10.7	11.0	9.8	8.5				

TABLE 3

Working surface illumination in L-shaped room on a clear day (in foot-candles = 10.76 lux) Sun polar angle: 45° off zenith; sun azimuth angle: 30° off South towards East. Illumination on outside horizontal surface: 6235 fc (67089 lux) direct sun, 1493 fc (16065 lux) sky component.

x	y: 8.75	6.25	3.75	1.25	−1.25	−3.75	−6.25	−8.75
−1.25	177.3	679.2	5837.2	272.4	279.4	934.4	5060.7	315.0
−3.75	216.4	358.3	432.3	366.4	334.3	478.7	524.5	405.1
−6.25	175.7	219.9	247.3	265.7	286.3	315.4	320.8	289.1
−8.75	132.3	147.3	163.8	188.9	238.8	248.7	244.3	224.5
−11.25	98.8	108.2	117.9	116.1				
−13.75	74.8	81.9	82.1	69.6				
−16.25	62.1	66.0	62.8	53.3				
−18.75	53.6	55.8	50.0	42.5				

the room in the x–z plane (thin line in Fig. 2). Thus comparison of the first (y = 8.75) column with the fourth (y = 1.25), the second with the third, etc., shows the influence of the L-shape of the room with its two windows. For the front half of the room, illumination at y = 1.25 is higher than at y = 8.75 because of its proximity to the second window. In the rear half of the room, however, the opposite is true as there is no direct illumination from the second window at y = 1.25. Comparison of the left-front third of the room with the right-front third shows the influence of the partly diffusing nature of the second window: close to the partly diffuse window the illumination is roughly 20% lower than close to the clear window (α = 0.8!). However, farther away from the windows, say along x = −8.75, illumination is higher in the right part of the room due to the diffusing nature of its window. Similar trends can be observed from the data for a clear day shown in Table 3. Note that only two of the shown nodes receive direct sunshine, *viz.*, x = −1.25, y = −3.75 and y = −6.25. This is due to the visual obstruction caused by the thickness of the window wall. For a zero wall thickness four nodes for each window would receive direct sun. To permit better visualization of the illumination levels in a room, a contour plotting routine has been added to the computer program (Fig. 3). Contour levels can be expressed in footcandles (lux) or in units of daylight factor. The plot shows the room outline, window location, and identifies the location of direct sunlight on the workplace.

Results for the A-frame room are summarized in Table 4 (overcast day) and Table 5 (clear day). For the overcast day, cabin geometry as well as sky luminance are symmetric, resulting in double symmetric illumination on the working surface (around x = 7.5 and around y = −12.5). In some cases the symmetry does not carry all the way to the last digit due to the iterative nature of the

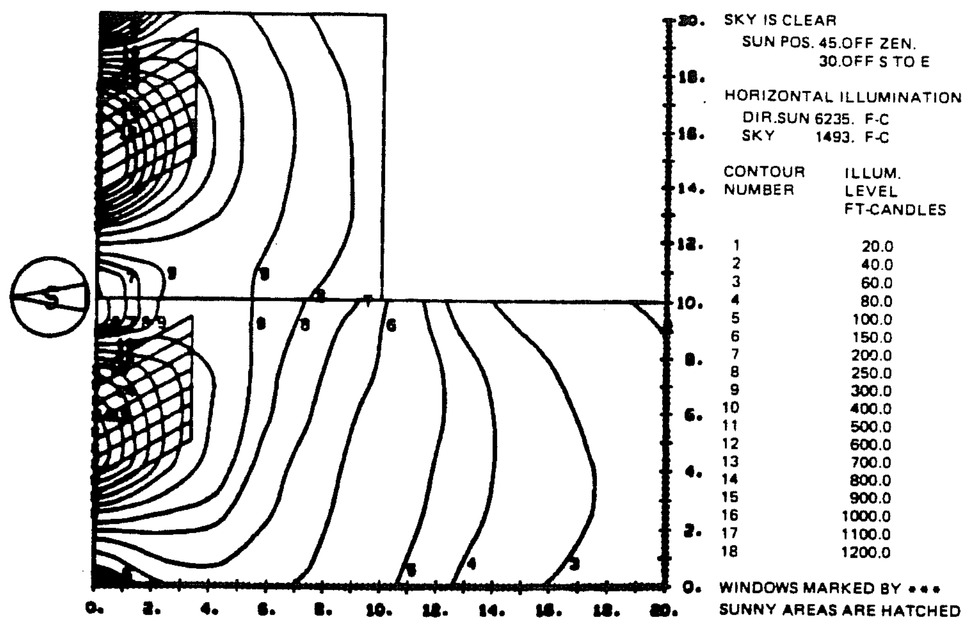

Fig. 3. Sample of graphical display of results for L-shaped room.

TABLE 4

Overcast-day daylight illumination on a working surface in A-frame cabin (in fc = 10.76 lux) (Illumination on horizontal outside surface = 1500 fc = 16140 lux.)

x	y: −1.25	−3.75	−6.25	−8.75	−11.25	−13.75	−16.25	−18.75	−21.25	−23.75
12.5										
(0)*	262.9	157.2	108.0	81.6	68.3	68.3	81.6	108.0	157.2	263.0
(1)	228.0	138.6	93.4	70.0	59.1	59.1	70.0	93.4	138.7	228.0
(3)	92.5	89.6	67.2	51.2	42.3	42.3	51.2	67.2	89.7	92.6
10.0										
(0)	373.7	238.4	145.7	100.0	80.2	80.2	100.0	145.7	238.5	373.8
(1)	311.0	184.8	115.9	82.5	67.3	67.3	82.5	115.9	184.8	311.1
(3)	220.9	120.2	81.4	56.3	45.6	45.6	56.3	81.4	120.3	221.0
7.5										
(0)	385.9	266.7	161.2	106.8	84.6	84.6	106.8	161.3	266.8	386.0
(1)	336.5	208.5	128.3	84.6	68.4	68.4	84.6	128.3	208.5	336.6
(3)	190.9	138.4	84.7	59.5	47.1	47.1	59.6	84.7	138.4	191.0
5.0										
(0)	373.7	238.4	145.7	100.0	80.2	80.2	100.0	145.7	238.5	373.8
(1)	311.0	184.8	115.9	82.5	67.3	67.3	82.5	115.9	184.8	311.1
(3)	220.9	120.2	81.4	56.3	45.6	45.6	56.3	81.4	120.3	221.0
2.5										
(0)	262.9	157.2	108.0	81.6	68.4	68.4	81.7	108.0	157.2	263.0
(1)	228.0	138.6	93.4	70.0	59.1	59.1	70.0	93.4	138.7	228.1
(3)	92.5	89.6	67.2	51.2	42.3	42.3	51.2	67.2	89.7	92.6

***(0) = 0 ft window overhang, $\tau \neq$ const.; (1) = 1 ft, $\tau \neq$ const.; (3) = 3 ft, $\tau \neq$ const.**

solution. The results are shown for three different constructions: (0) no window overhangs, (1) 1 ft (0.30 m) wide, and (3) 3 ft (0.91 m) wide roof overhangs on both sides of the room. The results clearly demonstrate the strong influence that window overhangs or even the window wall thickness can have on daylighting. Along the centerline of the room the light level is essentially halved by the addition of 3 ft (0.9 m) overhangs. The effect of overhangs as shown here is slightly exaggerated, as it was assumed that the undersides

TABLE 5

Clear-day daylight illumination on working surface in A-frame cabin (in fc = 10.76 lux)
(Illumination on horizontal outside surface = 6106 fc (65701 lux) direct component, = 894 fc (9619 lux) sky component)

x	y: −1.25	−3.75	−6.25	−8.75	−11.25	−13.75	−16.25	−18.75	−21.25	−23.75
12.5										
(0_c)*	253.6	184.3	136.4	111.8	92.1	85.1	86.6	96.2	122.8	141.8
(0)	220.0	165.6	126.1	104.8	87.3	81.1	82.5	90.5	111.0	118.9
(1)	203.1	153.7	115.3	95.1	78.8	72.9	73.7	80.9	101.4	107.2
(3)	68.1	91.0	76.2	64.4	54.6	52.2	54.9	60.1	69.0	55.0
10.0										
(0_c)	5670.2	305.1	204.5	153.6	121.5	109.1	110.9	125.6	161.6	204.0
(0)	4760.1	279.4	190.5	144.3	115.2	103.9	105.5	118.5	148.1	166.8
(1)	341.2	224.8	162.6	127.9	102.6	92.2	92.5	101.8	124.1	140.5
(3)	199.3	134.1	102.8	80.6	66.5	62.2	64.5	73.0	86.6	94.8
7.5										
(0_c)	5735.3	409.3	263.1	181.2	138.0	120.1	120.6	138.0	175.7	219.7
(0)	4802.1	372.8	245.8	171.0	131.1	114.5	114.8	130.3	161.7	174.6
(1)	4761.0	309.4	208.2	145.0	113.2	99.1	98.0	111.1	136.2	151.1
(3)	218.0	175.8	121.9	92.3	74.2	68.4	70.0	77.6	94.5	87.6
5.0										
(0_c)	5810.0	5689.5	285.2	190.8	138.2	118.0	116.3	131.0	171.5	219.0
(0)	4862.9	4801.9	266.3	180.8	132.0	113.0	111.1	123.8	156.6	174.1
(1)	4825.0	4742.8	225.8	155.5	115.0	98.5	96.5	105.6	130.2	148.8
(3)	4688.4	231.3	135.0	98.0	74.6	67.2	67.4	75.6	89.8	101.9
2.5										
(0_c)	5814.0	5706.7	5457.0	170.4	121.0	101.5	97.1	105.8	138.1	172.7
(0)	4869.6	4805.5	4603.3	162.0	116.3	97.8	93.1	99.9	123.9	138.0
(1)	4837.8	4777.0	216.8	141.0	102.0	86.1	81.8	87.8	111.0	122.0
(3)	4689.4	309.3	149.3	96.8	70.5	62.0	60.4	65.0	74.6	65.3

*(0_c) = 0 ft window overhang, τ = const.; (0) = 0 ft overhang, $\tau \neq$ const.; (1) = 1 ft, $\tau \neq$ const.; (3) = 3 ft, $\tau \neq$ const.

of the overhangs are perfectly black (*i.e.*, have zero reflectivities). In reality there may be some light reflection from the overhangs into the room.

Again, the results for clear conditions exhibit the same trends. In Table 5 a fourth case, (0_c), is included (no overhangs, with directionally independent window transmissivities, τ = 85% = const.), to demonstrate the effect of directionally dependent transmissivities. Close to the windows the illumination is overpredicted by approximately 20% if τ = const. is used, as there light penetrates through the windows at nearly grazing angles corresponding to low glass transmissivities. The effect diminishes to roughly 5% near the center of the room where direct sky light penetrates through the windows at near-normal angles. The difference is mainly due to the fact that most glazing materials, such as glass with directional transmissivity, have a hemispherical transmissivity of less than its normal value (85%).

Future work will add two important modeling capabilities to the program. First, the program will be modified to account for the contribution of electric lighting fixtures. This will allow simulation of the total resultant illumination from daylight and electric lighting and permit investigations of changes in illumination distributions resulting from different electric lighting control strategies. Second, solution of the daylight contribution in a space from sunlit shading devices represents a mathematically intractable problem due to the geometrical complexities of many of the architectural shading solutions. However, these window systems can be modeled by the program if the angular distribution of the luminous intensity of the window/shading system is known. The luminous properties of shading systems will be empirically measured and then used as an input to a modified version of this program. With these capabilities added, the program will be able to model a broad range of daylighting and electric lighting conditions in realistic architectural spaces.

CONCLUSIONS

A general analytical model and numerical code have been developed that can accurately and efficiently evaluate daylighting for a vast variety of room designs. Computer time requirements for the sample cases discussed above were generally below ten seconds on an IBM 3033. The analytical model is capable of treating complicated design features such as sheer curtains, window overhangs, non-rectangular windows, cathedral ceilings, internal partial obstructions, etc. Thus, it is felt that the model can be of valuable help for daylighting considerations in the vast majority of practical building designs.

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